# Calculations of Ground- and Excited-State Potential Surfaces for Conjugated Heteroatomic Molecules 

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#### Abstract

The QCFF/PI method is extended to conjugated heteroatomic molecules. The extension is based on a new formalism for $\sigma-\pi$ separability which starts from the all-valence-electron Hamiltonian, fixes the $\sigma$-electron charge distribution, and introduces it as an effective potential in the Hamiltonian of the $\sigma$ electrons. The semiempirical parameters are obtained by fitting calculated and observed atomization energies, ionization energies, excitation energies, equilibrium geometries, dipole moments, and vibrational frequencies of nitrogen- and oxygen-containing conjugated molecules. The method provides a useful tool for calculating equilibrium conformations, electronic spectra, and resonance Raman spectra of large biological chromophores and for studying the effect of solvent or protein environments on such properties.


## I. Introduction

Conjugated molecules containing nitrogen and oxygen "heteroatoms" are important parts of most biological chromophores, including such molecules as porphyrins, chlorophylls, nucleic acids, retinal, and flavins. Detailed theoretical studies of such systems might help in understanding more deeply the mechanism of important biological processes. Thus, for example, there is currently significant interest in interpretation of resonance Raman (RR) spectra of biological molecules, ${ }^{1-3}$ in the effect of proteins on the absorption spectra of their prosthetic groups, ${ }^{4-6}$ and in the factors that control electron transfer in proteins. ${ }^{7,8}$ Studies of such problems require methods capable of reliable evaluation of ground- and excited-state equilibrium geometries and vibrational frequencies. Such methods should be flexible enough to allow incorporation of the effects of solvent and/or protein environment. While ab initio quantum mechanical approaches can treat small heteroatom molecules, they are too expensive to be used for detailed studies of large conjugated molecules. It is possible to use all-valence-electron semiempirical approaches (e.g., extended Hückel, ${ }^{9}$ CNDO, ${ }^{10-12}$ INDO, ${ }^{13}$ PCILO, ${ }^{14}$ PRDDO, ${ }^{15}$ and MINDO ${ }^{16}$ ). However, although very promising and very useful in many cases, these methods have not been developed to the degree of reliability and efficiency needed for calculations of ground- and excited-state equilibrium geometries and vibrational frequencies of large molecules. The other possibility is to assume $\sigma-\pi$ separability and represent the $\sigma$ potential surface by empirical potential functions and the $\pi$ electron potential surfaces by a semiempirical approach. Many calculations of these types have been performed (see, for example, ref 17 and 18) and considerable success has been achieved in

[^0]evaluation of electronic properties of rigid molecules. In general, the potential surfaces provided by these methods do not allow detailed conformational and vibrational studies.

A possible approach for detailed studies of large conjugated molecules is the quantum mechanical extension of the consistent force field to conjugated molecules (QCFF/PI). ${ }^{19-21}$ This method is based on a formal separation of $\sigma$ and $\pi$ electrons, with the former represented by analytical empirical potential functions and the latter by a second-order analytical representation of a semiempirical model of the Pariser-Parr-Pople type corrected for orbital overlap. The method provides both ground- and excit-ed-state potential surfaces for large conjugated molecules, allowing one to evaluate many molecular properties including equilibrium geometries, vibrational frequencies, and resonance Raman intensities. ${ }^{20}$ In view of the previous success of the QCFF/PI method in treating conjugated hydrocarbons, it seems quite natural to extend this method to conjugated heteroatomic molecules. The basic problem is that in heteroatom systems the $\sigma$-electron core is no longer constant; the changes in the $\sigma$ charges strongly influence the local potentials of the $\pi$ electrons. Thus, in order to retain the advantages of the QCFF/PI approximation one must reformulate the $\sigma-\pi$ separability approach. This is done here using a new approach based on a formal all-valence-electron treatment in which the $\sigma$-electron densities are evaluated empirically and constitute the core potential for the $\pi$-electron calculations, while the back polarization of the $\sigma$ electrons by the $\pi$ electrons is neglected.
Section II describes the reformulation of the $\sigma-\pi$ separability approximation, Section III considers the refinement of the energy parameters for heteroatomic molecules and compares the final refined set of calculated properties with observed properties. Section IV describes several applications of the method including calculations of resonance Raman spectra of porphyrins and environmental effects on absorption spectra of chromophores in proteins.

## II. Theoretical Approach

(a) Reformulation of the $\sigma-\pi$ Separability Approximation. In the case of conjugated hydrocarbons the QCFF/PI potentials surface of the $N$ th electronic state is given by

$$
\begin{equation*}
V^{N}(\mathbf{r})=V_{\sigma}{ }^{0}(\mathrm{r})+V_{\pi}^{0}(\mathbf{r})+\Delta V_{\pi}^{N}(\mathbf{r}) \tag{1}
\end{equation*}
$$

where $\mathbf{r}$ is the Cartesian coordinate vector, $V_{\sigma}{ }^{0}(\mathbf{r})+V_{\pi}{ }^{0}(\mathbf{r})$ is the sum of the $\sigma$ - and $\pi$-electron energies for the gound state, and $\Delta V_{\pi}^{N}(\mathbf{r})$ is the $\pi$-electron ${ }^{20}$ excited energy. The surface $V_{\sigma}(\mathbf{r})$ is represented by empirical potential functions. The evaluation of the $\pi$ surface is based on a SCF-MO CI method. ${ }^{20}$ In this method

[^1]it is assumed that the $\sigma$-electron distribution is identical for all carbon atoms. Conjugated molecules with O and N heteroatoms require a more sophisticated approach since the $\sigma$ electrons may be strongly polarized and this will alter the corresponding $\pi$ electron core potentials. In order to extend the QCFF/PI approach to heteroatom systems, we reformulated the $\pi$-electron treatment to include the interaction with the $\sigma$ electrons while retaining the $\sigma-\pi$ separability assumption. Our treatment is based on considering the all-valence-electron Hamiltonian, fixing the $\sigma$-electron distribution, and constructing an effective $\pi$-electron Hamiltonian. The starting LCAO wave function is given by
\[

$$
\begin{equation*}
\Phi_{n}=\sum_{\mu} \mathbf{v}_{n \mu}{ }^{\boldsymbol{\pi}} \chi_{\mu}{ }^{\pi}+\sum_{\mu} \mathbf{v}_{n \nu}{ }^{\sigma} t_{\nu}{ }^{\sigma} \tag{2}
\end{equation*}
$$

\]

where the $\chi_{\mu}{ }^{\pi}$ 's are $2 \mathrm{p}_{z}$ Slater atomic orbitals, and the $t_{\nu}{ }^{\sigma}$ 's are $\mathrm{sp}^{2}$ hybrid $\sigma$ orbitals. Following ref 19 we actually work with Löwdin orbitals. (The index $\lambda$ is omitted for simplicity.) It is assumed formally that the $t^{\sigma}$ and $\chi^{\pi}$ orbitals are orthogonal. The coefficients $\mathbf{v}_{n}{ }^{\boldsymbol{\pi}}$ and $\mathbf{v}_{n}{ }^{\boldsymbol{\sigma}}$ are obtained by solving the all-valenceelectron SCF equation.

$$
\begin{equation*}
\mathrm{Fv}_{n}=\epsilon_{n} V_{n} \tag{3}
\end{equation*}
$$

The appropriate $\mathbf{F}$ matrix is of the following form:

$$
\left|\begin{array}{ll}
\mathbf{F}^{\pi} & \mathbf{F}^{\pi-\sigma}  \tag{4}\\
\mathbf{F}^{\pi-\sigma} & \mathbf{F}^{\sigma}
\end{array}\right|
$$

The assumption of $\sigma-\pi$ orthogonality implies that $\mathbf{F}^{\pi-\sigma}=0$ and yields two sets of solutions to the SCF equation, $\mathbf{v}_{n}{ }^{\pi}=\left(\mathbf{v}_{n}{ }^{\pi}, \mathbf{0}\right)$ and $\mathbf{v}_{m}{ }^{\boldsymbol{\sigma}}=\left(\mathbf{0}, \mathbf{v}_{m}{ }^{\boldsymbol{\sigma}}\right.$ ) (all the $\mathbf{v}_{n \nu}{ }^{\boldsymbol{\sigma}}$ are zero for $\Phi_{n}{ }^{\boldsymbol{\pi}}$ and all the $\mathbf{v}_{n \mu}{ }^{\boldsymbol{\pi}}$ are zero for $\Phi_{n}{ }^{\sigma}$ ).

Assuming that the complete $\mathbf{F}$ matrix is treated with a CNDO-type all-valence-electron approximation, we obtain (in the notation of ref 11 ):

$$
\begin{align*}
& F_{\mu \mu}=U_{\mu \mu}+\left(P_{\mathrm{A}}-1 / 2 P_{\mu \mu}\right) \gamma_{\mathrm{AA}}+ \\
& \sum_{\mathrm{B} \neq \mathrm{A}}^{\text {atoms with }} \begin{array}{c}
\pi \text { electrons } \\
\text { atoms with only } \\
\sigma \text { electrons } \\
\left.\sum_{\mathrm{B}} \gamma_{\mathrm{AB}}-V_{\mathrm{AB}}\right)+\sum_{\mathrm{C} \neq \mathrm{A}}\left(P_{\mathrm{C}} \gamma_{\mathrm{AC}}-V_{\mathrm{AC}}\right) \\
F_{\mu \nu}=\beta_{\mu \nu}-1 / 2 P_{\mu \nu} \gamma_{\mathrm{AB}}
\end{array}
\end{align*}
$$

where the $\mu$ th orbital is attached to the Ath atom, $U$ is the resonance integral, $\gamma$ is the two-electron repulsive integral, $V$ is the penetration integral, $P$ is the bond order, and

$$
\begin{equation*}
P_{\mathrm{A}}=\left(\sum_{\nu}^{\substack{\text { all } \sigma \text { orbitals } \\ \text { on } \mathrm{A}}} P_{\nu v}\right)+P_{\mathrm{A}}^{\pi}=P_{\mathrm{A}}^{\pi}+P_{\mathrm{A}}^{\sigma} \tag{7}
\end{equation*}
$$

Next, we modify $\mathbf{F}_{\mu \mu}{ }^{\boldsymbol{x}}$ to a form similar to the one used in $\pi$-electron calculations. This is done by the following: (i) defining for each atom a total core charge $\bar{Z}=Z^{\boldsymbol{\pi}}+Z^{\sigma}$ as the number of valence electrons where $Z^{\boldsymbol{x}}$ is taken arbitrarily as 1 for $\mathrm{C}, \mathrm{O}$, and pyridine nitrogen and 2 for pyrrole-type nitrogen; (ii) defining the net $\sigma$ and $\pi$ charges by $Q_{\mathrm{A}}{ }^{\sigma}=\left(Z_{\mathrm{A}}{ }^{\sigma}-P_{\mathrm{A}}{ }^{\sigma}\right)$ and $Q_{\mathrm{A}}{ }^{\pi}=\left(Z_{\mathrm{A}}{ }^{\pi}\right.$ $-P_{\mathrm{A}}{ }^{\pi}$ ), assuming $Q^{\sigma}$ is constant (as implied by solving $\mathbf{F}^{\sigma-\sigma}$ first and then keeping the resulting $\pi$ bond order constant); (iii) using the usual approximation $V_{\mathrm{AB}}=Z_{\mathrm{B}} \gamma_{\mathrm{AB}}$, and defining $W_{\mu}=\left(U_{\mu \mu}\right.$ $\left.+Z_{\mathrm{A}}{ }^{\sigma} \gamma_{\mathrm{AA}}\right)$. The diagonal element of $\mathbf{F}^{\pi}$ becomes


The off-diagonal elements of $F^{\boldsymbol{x}}$ are not modified.
Three different components are present in the final $F_{\mu \mu}$ : (i) the "effective" core of a $\pi$ electron on atom A which includes the $Q^{\sigma}$ charge contribution, (ii) the usual $\pi-\pi$ electron interaction term, and (iii) the contribution arising from the interaction between a $\pi$ electron on atom $A$ and the electric field generated by the " $\sigma$-core" charges of the surrounding atoms. Defining the "effective" core ( $W_{\mu}$ ) as $U_{\mu \mu}+Z_{\mathrm{A}}{ }^{\sigma} \gamma_{\mathrm{AA}}$ is particularly useful for determining the parameters of different states of the same atomic species. For example, the nitrogen atom in pyrrole $\left(Z^{\pi}=2\right)$ and
pyridine ( $Z^{\kappa}=1$ ) compounds will have the same $U_{\mu \mu}$, but their core will differ by $\gamma_{\text {AA }}$ because their values for $Z^{\sigma}$ differ ( $Z^{\sigma}=$ 4 for pyridine; $Z^{\sigma}=3$ for pyrrole).
In the present approximation the total energy can be expressed as:

$$
\begin{equation*}
E=\sum_{\mathrm{A}} E_{\mathrm{A}}+\sum_{\mathrm{A}>\mathrm{B}} E_{\mathrm{AB}} \tag{9}
\end{equation*}
$$

where $E_{\mathrm{A}}$ and $E_{\mathrm{AB}}$ are given by:

$$
\begin{gather*}
E_{\mathrm{A}}=P_{\mathrm{A}}{ }^{\pi}\left(W_{\mu}-1 / 2 Q_{\mathrm{A}}{ }^{\sigma} \gamma_{\mathrm{AA}}\right)+1 / 4\left(P_{\mathrm{A}}^{\pi}\right)^{2} \gamma_{\mathrm{AA}}+E_{\mathrm{A}}^{\sigma}  \tag{10}\\
E_{\mathrm{AB}}=\sum_{\mu} \sum_{\nu}\left[2 P_{\mu \nu}{ }^{\pi} \beta_{\mu \nu}-1 / 2\left(P_{\mu \nu}{ }^{\pi}\right)^{2} \gamma_{\mathrm{AB}}\right]+ \\
\left(Q_{\mathrm{A}}{ }^{\pi}+Q_{\mathrm{A}}^{\sigma}\right)\left(Q_{\mathrm{B}}{ }^{\pi}+Q_{\mathrm{B}}^{\sigma}\right) \gamma_{\mathrm{AB}}+E_{\mathrm{AB}}{ }^{\sigma}
\end{gather*}
$$

$\mu$ and $\nu$ are on atoms A and B, respectively, and the terms $E_{\mathrm{A}}{ }^{\sigma}$ and $E_{\mathrm{AB}}{ }^{\sigma}$ are defined as

$$
\begin{gather*}
E_{\mathrm{A}}{ }^{\sigma}=\sum_{\mu}^{\substack{\sigma \text { orbitals } \\
\text { on } \mathrm{A}}} P_{\mu \mu}{ }^{\sigma} U_{\mu \mu}+1 / 2 \sum_{\mu}^{\substack{\sigma \text { orbitals } \\
\text { on } \mathrm{A}}} \sum_{\nu}\left(P_{\mu \mu}{ }^{\sigma} P_{\nu \nu}{ }^{\sigma}-1 / 2\left(P_{\mu \nu}{ }^{\sigma}\right)^{2}\right) \gamma_{\mathrm{AA}}  \tag{11}\\
E_{\mathrm{AB}}{ }^{\sigma}=\sum_{\mu}^{\sigma \text { orbitals }} \begin{array}{r}
\sigma \text { orbitals } \\
\text { on }
\end{array} \sum_{\nu}^{\text {on } \mathrm{B}}\left(2 P_{\mu \nu}{ }^{\sigma} \beta_{\mu \nu}-1 / 2\left(P_{\mu \nu}{ }^{\sigma}\right)^{2} \gamma_{\mathrm{AB}}\right)
\end{gather*}
$$

These terms represent the energy due to the $\sigma$ - electron interaction. This energy is absorbed in the empirical potential functions of the $\sigma$ - potential surface.
(b) QCFF/PI Calculations for Heteroatom Systems. Using the formulation of the previous section, we perform the $\pi$-electron calculation by assigning a set of fixed $\sigma$ charges (that depend only on the bonding topology) and solving the SCF equation:

$$
\begin{equation*}
F^{\boldsymbol{x}} \mathrm{V}^{\boldsymbol{\pi}}=\epsilon \mathrm{V}^{\boldsymbol{\pi}} \tag{12}
\end{equation*}
$$

The $\mathbf{F}^{\boldsymbol{x}}$ matrix elements (eq 6 and 8) are given as

$$
\begin{gather*}
F_{\mu \nu}{ }^{\pi}=\bar{W}_{\mu}+\left(P_{\mathrm{A}}{ }^{\pi}-1 / 2 P_{\mu \mu}\right) \gamma_{\mathrm{AA}}-\sum_{\mathrm{B}} \bar{Q}_{\mathrm{B}} \gamma_{\mathrm{AB}}  \tag{13}\\
F_{\mu \nu}{ }^{\pi}=\beta_{\mu \nu}-1 / 2 P_{\mu \nu} \gamma_{\mu \nu}
\end{gather*}
$$

where $\bar{W}_{\mu}=W_{\mu}-K^{W} Q_{A}{ }^{\sigma} \gamma_{\mathrm{AA}}, \bar{Q}=Q^{\pi}+Q^{\sigma}$, B runs over all atoms, and $Q^{\pi}$ is zero for atoms without $\pi$ electrons. $\gamma_{A B}$ for interaction of $\pi$ atoms with $\sigma$ atoms is taken as $e^{2} / r . K^{W}$ in $W_{\mu}$ is a screening parameter ( $K^{W} \leq 1$ ) needed for simultaneous fitting of dipole moments and ionization potentials (see below). The functional form of the integrals $W_{\mu}, \beta$, and $\gamma$ are the same used in ref 19.

The $\pi$-electron ground-state potential surface is given by eq 9 and 10 , which can be rewritten as:

$$
\begin{align*}
& V_{\pi}^{0}(\mathrm{r})=\sum_{\mathrm{A}}\left[P_{\mathrm{A}}{ }^{\pi}\left(W_{\mu}-1 / 2 Q_{\mathrm{A}}{ }^{\sigma} \gamma_{\mathrm{AA}}\right)+1 / 4\left(P_{\mathrm{A}}{ }^{\pi}\right)^{2} \gamma_{\mathrm{AB}}\right]+ \\
& \sum_{\mathrm{A}>\mathrm{B}} \sum_{\mu} \sum_{\nu}\left[2 P_{\mu \nu}{ }^{\pi} \beta_{\mu \nu}-1 / 2\left(P_{\mu \nu}{ }^{\pi}\right)^{2} \gamma_{\mathrm{AB}}\right]+\sum_{\mathrm{A}>\mathrm{B}} \bar{Q}_{\mathrm{A}} \bar{Q}_{\mathrm{B}} \gamma_{\mathrm{AB}}+ \\
& \sum_{\mathrm{A}, \mathrm{C}}\left[\bar{Q}_{\mathrm{A}} Q_{\mathrm{C}}{ }^{\sigma}\right] e^{2} / r_{\mathrm{AC}} \tag{14}
\end{align*}
$$

where A and B run over $\pi$ atoms and C runs over $\sigma$ atoms. The last term represents the electrostatic interaction between atoms with $\pi$ electrons (A) and atoms without $\pi$ electrons (C). the bond orders, $P_{\mathrm{A}}{ }^{\pi}$, and the $\pi$ charges, $Q_{\mathrm{A}}{ }^{\pi}$, are evaluated by solving eq 12.

The $\sigma$-electron potential surface is given by:

$$
\begin{aligned}
& V_{\sigma}{ }^{0}(\mathbf{r})=\sum_{i} M\left(b_{i}\right)+1 / 2 \sum_{i}\left[K_{\mathrm{a}}\left(a_{\mathrm{i}}-a_{0}\right)^{2}+2 D_{i}\right]+1 / 2 \sum_{i} K_{\theta}\left(\theta_{i}-\right. \\
&\left.\theta_{0}\right)^{2}+1 / 2 \sum_{i} F\left(q_{i}-q_{0}\right)^{2}+1 / 2 \sum_{i} K_{\phi}{ }^{(1)} \cos \phi_{i}+1 / 2 \sum K_{\phi}{ }^{(2)} \cos 2 \phi_{i} \\
&+1 / 2 \sum_{i} K_{x}\left(\chi_{i}-\chi_{0}\right)^{2}+\sum_{i>j} f\left(r_{i j}\right)+\sum_{k>l}\left(Q_{k}{ }^{\sigma} Q_{1}{ }^{\sigma}\right) e^{2} / r_{k l}
\end{aligned}
$$

where $M$ is a Morse potential function, $b_{i}$ are bond lengths between atoms with $\pi$ electrons, and the $a_{i}$ are bond lengths between pairs of atoms which include at least one atom without $\pi$ electrons. $\theta_{i}$, $\phi_{i}$, and $\chi_{i}$ are respectively bond angles, torsion angles, and out-of-plane angles (see ref 20), $q_{i}$ are the 1-3 nonbonded distances, while $r_{i j}$ are nonbonded distances between atoms separated by more than one atom. The nonbonded function $f\left(r_{i j}\right)$ is the exp-6 function

Table I. Parameters for $\pi$-Electron Integrals ${ }^{a}$

${ }^{a}$ The functional form of the integrals $W, \beta$, and $\gamma$ as well as the notation of their different parameters is the same as in ref 20. The parameters for $C^{P} C^{P}$ are given in ref 20 . I-A is taken as the average of the $\mathrm{I}-\mathrm{A}$ of the corresponding isolated atoms. Unsaturated carbons are designated by $\mathrm{C}^{\mathrm{P}}$, pyrrole-like nitrogen $\left(-\mathrm{NH}-\right.$ ) by N , pyridine-like nitrogen ( $-\mathrm{N}-$ ) by $\mathrm{N}^{\mathrm{P}} . K^{W}$ is the screening parameter in $\bar{W}\left(\bar{W}=W-K^{W} Q_{\mathrm{A}} \sigma_{\gamma_{\mathrm{AA}}}\right)$.
used in ref 20. The last term represents the electrostatic interaction between all pairs of atoms which have no $\pi$ electrons (these include bonded atoms). The term $E_{\mathrm{A}}$ of eq 10 is not included as an atomic constant since it is effectively absorbed in the corresponding Morse potentials.

The treatment of excited $\pi$ potential surfaces is identical with that for hydrocarbon molecules, ${ }^{19}$ where the excitation energy $\Delta V_{\pi}^{N}(\mathbf{r})$ (eq 1) is obtained by the standard SCF-CI treatment using the modified $\mathbf{F}^{\boldsymbol{x}}$ matrix of eq 13 . Note that the present treatment does not evaluate $n \rightarrow \pi^{*}$ transitions.

The applications of the QCFF/PI potential surfaces to calculations of ground- and excited-state equilibrium geometries and vibrational frequencies are described in detail elsewhere. ${ }^{19,20}$ Here we apply the same procedure to the potential surfaces of conjugated heteroatomic molecules.
(c) Potential Parameters. The parameters of the $\pi$-electron integrals and the $\sigma$-potential functions are given in Tables I and II, respectively. These parameters were obtained by extensive least-squares fits of different calculated and observed properties (see next section). In addition to the above parameters the present treatment requires the $\sigma$ core charges ( $Q^{\sigma}$ ). These charges can be evaluated by an all-valence-electron approach. Here we evaluate the $\sigma$ charges by the method proposed by Del Re. ${ }^{22}$ In this method the net $\sigma$-charge on atom A is given by

$$
\begin{equation*}
Q_{\mathrm{A}}=\sum_{\mathrm{B}} q_{\mathrm{A}}{ }^{(\mathrm{B})} \tag{16}
\end{equation*}
$$

where B runs on all atoms connected to A and $q_{\mathrm{A}}{ }^{(\mathrm{B})}$ is the charge on atom A resulting from an $\mathrm{A}-\mathrm{B}$ covalent bond. $q_{\mathrm{A}}{ }^{(\mathrm{B})}$ is defined by

$$
\begin{equation*}
q_{\mathrm{A}}{ }^{(\mathrm{B})}=D_{\mathrm{A}}{ }^{(\mathrm{B})}\left(1+\left(D_{\mathrm{A}}{ }^{(\mathrm{B})}\right)^{2}\right)^{-1 / 2} \tag{17}
\end{equation*}
$$

where

$$
D_{\mathrm{A}}{ }^{(\mathrm{B})}=\left(\delta_{\mathrm{B}}-\delta_{\mathrm{A}}\right) /\left(2 \epsilon_{\mathrm{AB}}\right)
$$

The $\delta_{A}$ 's are computed by solving the following set of linear equations (for all atoms in the molecule)

$$
\begin{equation*}
\delta_{\mathrm{A}}=\delta_{\mathrm{A}}^{0}+\sum_{\mathrm{B}} \delta_{\mathrm{B}} \bar{\gamma}_{\mathrm{AB}} \tag{18}
\end{equation*}
$$

(22) G. Del Re, Theor. Chim. Acta, 1, 188 (1963).

Table II. Parameters for the $\sigma$ Potential Functions ${ }^{a}$

| bond | D | $\alpha$ | $1 / 2 K_{b}$ | $b_{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}^{\mathrm{P}}-\mathrm{N}$ | 65.0 | 2.10 |  | 1.340 |
| $\mathrm{C}-\mathrm{N}$ | 88.0 |  | 88.0 | 1.450 |
| $\mathrm{C}^{P}-\mathrm{N}^{P}$ | 65.0 | 2.00 |  | 1.255 |
| $\mathrm{N}-\mathrm{H}$ | 93.0 |  | 414.0 | 1.000 |
| $\mathrm{N}-\mathrm{N}^{\text {P }}$ | 40.0 | 2.90 |  | 1.309 |
| $\mathrm{N}^{\mathrm{P}}-\mathrm{N}^{\mathrm{P}}$ | 35.0 | 2.67 |  | 1.360 |
| $\mathrm{C}^{\mathrm{P}}-\mathrm{O}$ | 75.0 | 1.80 |  | 1.235 |
| bond angle | $1 / 2 K_{\theta}$ | $\theta$ 。 | 1/2F | $q_{0}$ |
| $\mathrm{C}^{P} C^{\text {P }}$ N | 58.3 | 2.094 | 432.4 | 2.56 |
| $\mathrm{C}^{\mathrm{P}} \mathrm{NC}^{\mathrm{P}}$ | 58.3 | 2.094 | 432.4 | 2.56 |
| $\mathrm{C}^{\text {P }} \mathrm{NH}$ | 24.0 | 2.094 | 429.5 | 2.18 |
| $\mathrm{NC}^{\mathbf{P}}{ }_{\mathrm{H}}$ | 24.0 | 2.094 | 429.5 | 2.18 |
| ${ }^{P}{ }^{\text {P }} \mathrm{C}^{\mathrm{P}} \mathrm{H}^{\text {P }}$ | 24.0 | 2.094 | 429.5 | 2.18 |
| $N^{P} \mathrm{P}^{P} \mathrm{C}^{\text {P }}$ | 52.8 | 2.094 | $4 \quad 29.5$ | 2.56 |
| $C^{P} N^{P}{ }^{\text {P }}{ }^{P}$ | 52.8 | 2.094 | 429.5 | 2.56 |
| $\mathrm{NC}^{P} \mathrm{~N}^{\mathrm{P}}$ | 52.8 | 2.094 | $4 \quad 32.4$ | 2.74 |
| ${ }^{( }{ }^{P} C^{P}{ }^{\text {P }}$ N ${ }^{\text {P }}$ | 52.8 | 2.094 | $4 \quad 32.4$ | 2.74 |
| $C^{P} N^{P}{ }_{N}{ }^{P}$ | 52.8 | 2.094 | 432.4 | 2.56 |
| torsion |  |  | torsion | $1 / 2 K_{\phi}$ |
| $\mathrm{X}-\mathrm{C}^{P}-\mathrm{N}-\mathrm{X}$ |  |  | $\mathrm{X}-\mathrm{N}-\mathrm{N}^{\mathrm{P}}-\mathrm{X}$ | 6.50 |
| $X-C^{P}-N^{P}-X$ |  |  | $\mathrm{X}-\mathrm{N}^{\mathrm{P}}-\mathrm{N}^{\mathrm{P}}$-X | 5.73 |
| out of plane |  |  | ${ }^{1 / 2} K_{\chi}$ |  |
|  | N | 3.0 |  |  |
| nonbonded ${ }^{\text {b }}$ | A |  | $B$ | $\mu$ |
| HN |  |  | 117 | 4.25 |
| HO |  |  | 79 | 4.19 |
| CN | 1189 |  | 718 | 4.31 |
| CO | 395 |  | 573 | 4.28 |
| NO | 399 |  | 562 | 4.28 |

${ }^{a}$ Units used are: energies in $\mathrm{kcal} / \mathrm{mol}$, lengths in angstroms, angles to radians; force constants are expressed correspondingly. ${ }^{b}$ The nonbonded potential functions used are $A \exp (-\mu r)-B r^{-6}$; the $\mathrm{C}-\mathrm{C}$ and $\mathrm{C}-\mathrm{H}$ parameters are as in ref 20 . The nonbonded parameters for N and $\mathrm{N}^{\mathrm{P}}$ are assumed to be equal.

Table III. Parameters for $\sigma$ Charges ${ }^{a}$

${ }^{a}$ Notation for atoms as in Table I.
The parameters $\delta_{\mathrm{A}}{ }^{0}, \bar{\gamma}_{\mathrm{AB}}$, and $\epsilon_{\mathrm{AB}}$ are given in Table III. These parameters were refined, together with all other energy parameters, by the least-squares fitting procedure.
(d) Possible Simplifications. The approach described in the previous sections, and used in the calculations reported in this paper, requires substantial modifications of the original QCFF/PI method. A less rigorous (and less reliable) treatment can be used in order to avoid modifications. It is possible to assign $\sigma$ charges

Table IV. Parameter Set for Simplified Calculations ${ }^{a}$

| atom | $\bar{W}$ | $\bar{Z}$ | $Q^{\sigma}$ |
| :--- | :---: | :--- | :--- |
| N | -15.1 | 0.90 | -0.15 |
| $\mathrm{~N}^{\mathrm{P}} \mathrm{H}$ | -28.6 | 1.80 | -0.20 |
| O | -17.3 | 0.90 | -0.10 |
| $\mathrm{C}^{\mathrm{P}}$ | -9.97 | $1.0+Q_{\mathrm{C}}{ }^{\sigma}$ | $\left(Q_{\mathrm{C}}{ }^{\sigma}\right)^{b}$ |

${ }^{a}$ All other parameters are taken without change from Tables I and II. ${ }^{b}$ The $\sigma$ charge of the carbon atoms is evaluated by taking $-Q^{\sigma}$ of the corresponding heteroatom and distributing it over the nearest neighboring carbon atoms. For example, in pyrrole the $\sigma$ charge of $\mathrm{C}_{5}$ and $\mathrm{C}_{2}$ is $(0.2) / 2=0.1$.
only on the basis of atom type, regardless of the bonding arrangement. In this case, the diagonal SCF matrix element are approximated by

$$
\begin{gather*}
\bar{W}_{\mu}=W_{\mu}-Q_{\mathrm{A}}{ }^{\sigma} \gamma_{\mathrm{AA}} \quad \bar{Q}_{\mu}^{\pi}=\left(\bar{Z}_{\mu}^{\pi}-P_{\mathrm{A}}{ }^{\pi}\right)  \tag{19}\\
\bar{Z}_{\mu}^{\pi}=Z_{\mu}^{\pi}+Q_{\mathrm{A}}{ }^{\sigma}
\end{gather*}
$$

This simplified treatment can be incorporated directly into the orginal QCFF/PI program. ${ }^{21}$ The corresponding parameters are given in Table IV.

The $\pi$-electron energy surface in the simplified approach is given by:

$$
\begin{align*}
V_{\pi}^{0}(\mathbf{r}) \simeq & \sum_{\mu} P_{\mu \mu}\left[\bar{W}_{\mu}+1 / 4 P_{\mu \mu} \gamma_{\mathrm{AA}}\right]+2 \sum_{\nu>\mu} P_{\nu \mu} \beta_{\mu \nu}- \\
& \sum_{\nu>\mu}\left[1 / 2 P_{\mu \nu}{ }^{2}-\bar{Q}_{\mu}{ }^{\pi} \bar{Q}_{\nu}{ }^{\pi}\right] \gamma_{\mu \nu}+\sum_{\mu, \mathrm{C}} \bar{Q}_{\mu}{ }^{\pi} Q_{\mathrm{C}} e^{2} / r_{\mu \mathrm{C}} \tag{20}
\end{align*}
$$

This energy expression which (excluding the last term) is the one used in the original QCFF/PI program gives reasonable equilibrium energies and relative energies, but unreliable atomization energies for heteroatom systems.

In general the simplified approach is recommended for qualitative studies while the nonsimplified approach (which was used in all the studies reported here) is recommended for quantitative studies.

## III. Determination of Semiempirical Parameters

Section II describes a procedure for calculating the total energy surface $V^{N}(\mathbf{r})$ (relative to the separated atoms) of a conjugated heteroatomic molecule in the $N$ th electronic state. Since the expression $V^{N}(\mathbf{r})$ involves a mixture of semiempirical concepts implemented in terms of rather complicated functions that depend on many parameters, the results can be regarded as meaningful only if they apply to a significant number of independent properties for a variety of molecules. Here we determine the parameters by the CFF least-squares procedure. ${ }^{20,23}$ This method determines a set of parameters which yield satisfactory agreement between the calculated and experimental properties depending on the zeroth, first, and second derivative terms in a Taylor expansion of the potential energy surfaces of various molecules. The effectiveness of the procedure is based on the availability of analytic expressions for the potential energy and its first and second derivatives with respect to the atomic Cartesian coordinates (see ref 20 for more details). A large number of experimental data were employed in the fitting procedure. They include: ionization energies, excitation energies, atomization energies, dipole moments, equilibrium conformations, and vibrational frequencies of conjugated molecules containing nitrogen and/or $\mathrm{C}=\mathrm{O}$ groups. The molecules included in this data base are shown in Figure 1. The next sections give additional information about the properties included in the least squares.

The nonbonded interaction parameters were determined by an independent procedure because the data included above are not sufficiently sensitive to these interactions. For $\mathrm{C}-\mathrm{H}$ and $\mathrm{C}-\mathrm{C}$ interactions we used the previous QCFF/PI parameters. ${ }^{20}$ For interactions involving N and O we employ a modified form of the potential functions of ref 37 ; that is, these potentials were fitted by exp-6 potentials over a series of distances and orientations. This

[^2]
PYRIDINE

QUINOLINE

ISOQUINOLINE

PYRAZINE

PYRIMIDINE

PYRIDAZINE

QUINOXALINE




F-BENZOQUNONE


BENZALDEHYOE


GUANINE


URACYL


ADENINE

Figure 1. Molecular diagrams of the molecules included in the QCFF/PI refinement procedure.
fitting involves a decrease of $0.1 \AA$ in all the $r^{*}$ of ref 37 to account for the difference between unit cell parameters at room temperature and at 0 K .
(a) Ionization Energies. The calculated ionization energies (obtained in the last iteration of the least-squares procedure) and the corresponding experimental results are given in Table V. The agreement between the calculated and observed values is good, and all the deviations are smaller than 0.6 eV . In some cases (e.g., the first ionization of pyridine) there is a large range of experimental values for a given property. In such cases we assign a low

Table V. Calculated and Observed Ionization Energies (in eV) Included in the Optimization of the QCFF/PF Parameters

| compound | calcd | obsd | compound | calcd | obsd |
| :---: | :---: | :---: | :---: | :---: | :---: |
| pyridine | 9.74 | $9.00,^{c} 9.28,{ }^{\text {b }} 9.31,{ }^{\text {g }} 9.70,^{\text {d }} 9.80^{\text {a }}$ | cinnoline | 9.22 | $8.51{ }^{\text {g,h }} 8.95^{\text {l }}$ |
|  | 10.12 | $10.45{ }^{\text {g }} 10.5,^{a} 10.5^{\text {c }}$ |  | 9.78 | 9.03g.h |
|  | 12.93 | $12.30{ }^{\text {g }} 12.6{ }^{\text {a }}$ |  | 11.05 | $10.83,{ }^{\text {g, }}$ h $9.75^{g}$ |
| quinoline | 8.73 | $8.62,{ }^{\text {g }} 8.67,^{m} 8.90^{d}$ |  | 12.31 | 12.04, ${ }^{\text {g, }}$ h $10.83^{g}$ |
|  | 9.23 | $9.00{ }^{m} 9.07^{\text {m }}$ |  | 14.00 | $13.85{ }^{\text {g }}$, ${ }^{\text {d }} 12.04^{g}$ |
|  | 10.67 | $10.63{ }^{\text {m }} 10.64^{\text {g }}$ | aniline | 7.9 | $7.71{ }^{\text {b }}$ |
|  | 11.60 | $11.37{ }^{\text {m }} 11.42^{\text {g }}$ |  | 9.1 | $8.95{ }^{\text {b }}$ |
|  | 13.22 | $13.07^{\mathrm{g}}$ |  | 10.9 | $10.49^{\text {b }}$ |
| isoquinoline | 8.66 | $8.53,{ }^{\text {m }} 8.54 \mathrm{~g}$ | formaldehyde | 12.9 | $13.99 .614 .09^{k}$ |
|  | 9.24 | 9.16, ${ }^{m} 9.24^{\mathrm{g}}$ | acetaldehyde | 12.6 | $12.75{ }^{\text {b }} 13.90^{\text {b }}$ |
|  | 10.52 | $10.32{ }^{\text {m }} 10.50^{\text {g }}$ | acetone | 12.3 | $12.16{ }^{\text {b }} 14.15^{\text {b }}$ |
|  | 11.73 | $11.43{ }^{\text {m }} 11.60^{\text {g }}$ | acrolein | 10.6 | $10.82,{ }^{\text {b }} 10.93^{k}$ |
|  | 13.20 | $13.26^{g}$ |  | 13.0 | $13.19{ }^{\text {b }} 13.20^{k}$ |
| pyrimidine | 10.24 | 9.35, ${ }^{\text {b }} 9.42,{ }^{\text {g }} 9.47,^{\text {b }} 9.91^{d}$ | glyoxal | 12.6 | $12.19^{\mathrm{k}}$ |
|  | 10.46 | $10.39{ }^{\text {g }} 10.39^{\text {b }}$ |  | 14.1 | $13.85{ }^{\text {k }}$ |
|  | 13.37 | $13.600^{\text {b }} 13.62^{\mathrm{g}}$ | $p$-benzoquinone | 10.4 | $10.49^{k}$ |
| pyridazine | 10.23 | $8.71,^{d} 8.90,^{\text {g }} 8.91,^{\text {b }} 9.86,{ }^{\text {d }} 10.15^{\text {c }}$ |  | 11.2 | $11.25^{k}$ |
|  | 10.79 | $10.533^{b} 10.55^{b}$ |  | 13.2 | $13.43^{k}$ |
|  | 13.85 | $13.59{ }^{\text {b }} 13.63^{g}$ | benzaldehyde | 9.7 | $9.46,{ }^{\text {i }} 9.51{ }^{\text {b }}$ |
| quinazoline | 9.73 | $9.02{ }^{\text {g }}$ |  | 9.9 |  |
|  | 10.99 | $10.72^{\text {g }}$ |  | 12.2 | $11.48^{i}$ |
|  | 12.01 | $12.02^{\mathrm{g}}$ | formamide | 10.6 | $10.13^{k}$ |
|  | 13.61 | $13.78{ }^{\text {g }}$ | pyrrole | 8.57 | $8.20,{ }^{b} 8.22,{ }^{\text {b }} 8.23,{ }^{\circ} 8.90,{ }^{d} 8.97{ }^{\text {d }}$ |
| quinoxaline | 9.67 | $9.02^{\text {g }}$ |  | 9.62 | $9.03,{ }^{\text {b }} 9.22^{\circ}$ |
|  | 11.25 | $10.72^{\text {g }}$ |  | 14.50 | $12.38{ }^{\circ}{ }^{\circ} 14.70^{\circ}$ |
|  | 11.90 | $11.58{ }^{\text {g }}$ | imidazole | 9.06 | $8.78{ }^{\circ}$ |
|  | 13.56 | $13.98{ }^{\text {g }}$ |  | 10.44 | $10.3{ }^{\circ}$ |
| phthalazine | 8.97 | $8.68{ }^{\text {h }} 8.68 \mathrm{~g}$ |  | 14.64 | $14.7{ }^{\circ}$ |
|  | 9.99 | 9.17, ${ }^{\text {¢ }} 9.17 \mathrm{~g}$ | pyrazole | 9.09 | $9.15{ }^{\circ}$ |
|  | 10.90 | 10.77, ${ }^{\text {h } 9.688}$ |  | 9.67 | $9.88^{\circ}$ |
|  | 12.37 | $12.07,{ }^{h} 10.778$ |  | 14.88 | $14.70^{\circ}$ |
|  | 13.84 | $14.09{ }^{\text {h }} 12.07 \mathrm{~g}$ |  |  |  |
| pyrazine | 10.19 | $9.27, b^{b .29, d, e} 9.16, g 9.62, f 10.01^{d}$ |  |  |  |
|  | 10.79 | $10.11, b 10.15, g^{\prime} 10.16^{d}$ |  |  |  |
|  | 13.23 | $13.10{ }^{\text {b }} 13.13,{ }^{\text {g }} 13.60^{f}$ |  |  |  |

${ }^{a}$ C. Batich, E. Heilbronner, V. Hornung, A. J. Ashe III, D. T. Clark, U. T. Cobely, D. Kilcast, and I. Scanlan, J. Am. Chem. Soc., 95,928 (1973). b D. W. Turner, Adv. Phys. Org. Chem., 4, 31 (1966). c M. N. Pisanias, L. G. Christophorou, J. G. Carter, and D. L. McCorkle, J. Chem. Phys., 58, 2110 (1973). d R. P. Blaunstein and L. G. Christophorou, Radiat. Res. Rev., 3, 69 (1971). e J. E. Parkin and K. K. Innes, J. Mol. Spectrosc., 15, 407 (1965). f J. H. D. Eland, Int. J. Mass Spectrom. Ion Phys., 2, 471 (1969). g M. J. S. Dewar and S. C. Worley, J. Chem. Phys., 51, 263 (1969). ${ }^{h}$ M. Sundbom, Acta Chem. Scand., 25, 487 (1971). ${ }^{i}$ K. Watanabe, J. Chem. Phys., 25, 542 (1957). ${ }^{j}$ S. Cradock, R. H. Findlay, and M. H. Palmer, Tetrahedron, 29, 2173 (1973). ${ }^{k}$ D. W. Turner, "Molecular Photoelectron Spectroscopy", Wiely, New York, 1970. ${ }^{l}$ J. H. D. Eland and C. J. Danby, Z. Naturforsch., Teil A, 23, 335 (1968). m A. J. Yencha and M. A. El-Sayed, J. Chem. Phys., 48, 3469 (1968).
weight ( $P$ in eq 12 of ref 20 ) for the particular observable in the least-squares refinement procedure. In the case of cinnoline and phthalazine the calculations support the assignment of Sundbom ${ }^{18}$ rather than that of Dewar and Worley. ${ }^{24}$

It appears that the high ionization potentials, which correspond to orbitals with major components on nitrogen atoms, are very useful in providing information about the corresponding $W$ and $\gamma$. In order to obtain a reasonable fit of the calculated and observed values of these ionization energies, while retaining a reasonable agreement between the calculated and observed dipole moments, we had to introduce the screening parameter $K^{W}$ (see Table I) for oxygen and pyridine nitrogen. This might reflect the effect of the lone-pair electrons.

It is encouraging to note that the carbonyl compounds reproduce nicely the experimental trend of the ionization potentials in formaldehyde, acetaldehyde, and acetone where the decrease in the energy of the occupied orbitals was reached by means of a purely $\sigma$ charge effect. This effect is due to the $\sigma$ charge donation from the methyl groups that make the unsatuated carbon progressively more negative, and therefore decrease its affinity to the $\pi$ electrons.
(b) Excitation Energies. The refined calculated excitation energies and the corresponding observed values are given in Table VI. The table also compares the calculated and observed oscillator stengths. The agreement between the calculated and observed results for the nitrogen compound is quite satisfactory. The

[^3]refinement procedure for the oxygen-containing compounds involved a large statistical weight for molecules with a single nonconjugated $\mathrm{C}=\mathrm{O}$ bond. This resulted in a significant discrepancy between the calculated and observed excitation energies of glyoxal. This inconsistency cannot be resolved in the present level of approximations.
(c) Atomization Energies. The calculated atomization energy is given by the difference between the molecular energy of eq 10 and the energies of the isolated atoms. The energy of an isolated atom does not include the term $E_{\mathrm{A}}{ }^{a}$ of eq 10 since this term is incorporated in the Morse potential.

The refined set of atomization energies and the corresponding observed values are given in Table VII. The atomization energies depend strongly on the diagonal $\pi$ bond orders which are very sensitive to the $\sigma$ charges. As the $\sigma$ charges are also important for the dipole moment and ionization energies, our fitting procedure represents a compromise. However, it should be noted that the present agreement between the calculated and observed atomization energies is similar to that obtained in all-valenceelectron calculations. ${ }^{12,16}$
(d) Dipole Moments. The final set of calculated dipole moments and the corresponding observed values are given in Table VIII. A negative correlation was found between the calculated magnitude of the dipole moments and the calculated ionization energies. A similar but less pronounced behavior appears in the correlation between the calculated dipole moments and the calculated atomization and excitation energies. Thus, it appears that good calculated dipole moment could be obtained only at the expense of poorer agreement in other properties. Since our main

Table VI. Calculated and Observed Excitation Energies (in eV) Included in the Refinement of the QCFF/PI Parameters (Oscillator Strength in Parentheses)

| compound | calcd | obsd | compound | calcd | obsd |
| :---: | :---: | :---: | :---: | :---: | :---: |
| pyridine | 4.91 (0.02) | 4.75-4.97(0.04) ${ }^{\text {a,i,l }}$ | carbazole | 4.58 (0.02) | 3.71-4.24 ${ }^{j, x}$ |
|  | 5.93 (0.22) | 6.17-6.37 (0.18) |  | 4.85 (0.33) | 4.27-4.85 |
|  | 6.54 (1.03) | 6.70-7.13 (0.66) |  | 5.31 (0.51) | 4.81-5.29 |
|  | 6.93 (0.80) | 7.10 |  | 5.65 (1.23) | 5.29-5.88 |
| quinoline | 4.20 (0.04) | $3.95(0.02)^{f, l, p}$ |  | 5.73 (0.84) | 5.95-6.15 |
|  | 4.60 (0.18) | 4.46-4.62 (0.12) |  | 6.03 (0.02) |  |
|  | 5.66 (1.71) | 5.35-5.50 (0.54) | imidazole | 5.94 (0.36) | $5.90^{\text {w }}$ |
|  | 5.82 (0.46) | 6.08-6.11 (0.93 | aniline | 4.6 (0.05) | $4.4(0.33)^{k, g g}$ |
| isoquinoline | 4.26 (0.03) | $3.86-4.03(0.02)^{b, c, q, l}$ |  | 5.5 (0.18) | 5.4 (0.14) |
|  | 4.52 (0.26) | 4.58-4.66 (0.11) |  | 6.2 (0.86) | 6.3 (0.51) |
|  | 5.64 (0.56) | 5.35-5.39 |  | 6.3 (1.05) | 6.8 (0.57) |
|  | 5.76 (1.13) | 5.70-5.75 (1.02) |  |  | 7.8 (0.68) |
|  | 5.92 (0.69) | 5.86-6.32 (s) | purine | 4.32 (0.17) | $4.71{ }^{\text {b }}$ b,cc |
| pyrimidine | 4.83 (0.00) | 5.00-526 (0.05) ${ }^{a, d, l, m}$ |  | 5.31 (0.33) | 5.10 |
|  | 6.12 (0.56) | 6.45-6.71 (0.16) |  | 6.08 (1.00) | 6.10 |
|  | 6.81 (1.04) | 6.89-7.44 (0.43) |  | 6.24 (0.51) | 6.60 |
|  | 6.99 (0.42) | 7.25-7.68 | formaldehyde | 8.04 (0.47) | $7.97{ }^{z}$ |
| pyrazine | 5.30 (0.02) | 4.49-4.93 (0.10) ${ }^{\text {a,d, }, l, m}$ | acetaldehyde | 8.03 (0.47) | $7.52^{z, e e, f f}$ |
|  | 5.85 (0.24) | 6.06-6.59 (0.14) | acetone | 8.08 (0.47) | $8.05^{a a}$ |
|  | 6.87 (1.00) | 7.15-7.67 (vs) | acrolein | 5.79 (0.67) | $5.89{ }^{z}$,hh |
| pyridazine | 5.14 (0.00) | 4.90-5.13 (0.02) ${ }^{\text {a,d,m }}$ | glyoxal | 6.41 (0.86) | $7.44{ }^{\text {y }}$ |
|  | 5.01 (0.33) | 6.15-6.67 (0.01) | benzaldehyde | 4.53 (0.02) | $4.52(0.02)^{d d}$ |
|  | 7.03 (0.41) | $7.10-7.33$ (1.0) |  | 5.15 (0.44) | 5.35 (0.26) |
| phthalazine | 4.39 (0.00) | $4.06-4.27(0.01)^{b, q, r, s}$ |  | 5.90 (0.55) | 6.35 (1.7) |
|  | 4.79 (0.20) | 4.64-4.78 (0.10) |  | 6.22 (0.84) | 6.68 (1.7) |
|  | 6.05 (1.56) | 5.78-5.86 (1.18) |  | 6.37 (0.18) | 6.97 (1.7) |
| cinnoline | 4.31 (0.01) | 3.91-3.93 ${ }^{l, t, x}$ |  | 7.12 (0.30) | 7.50 (1.7) |
|  | 4.33 (0.35) | 4.37-4.65 | $p$-benzoquinone | 3.7 (0.00) | $4.28(0.01)^{h}$ |
|  | 6.04 (1.97) | 5.49-5.65 |  | 4.6 (0.80) | 5.17 (0.15) |
|  | 6.42 (0.25) | $6.28$ | $o$-benzoquinone | 3.59 (0.2) | $3.3(0.66)^{h}$ |
| quinazoline | 4.20 (0.02) | $3.99-4.05(0.03)^{l, v}$ |  | 5.29 (0.2) | 4.9 (0.26) |
|  | 4.67 (0.27) | 4.56-4.65 (0.09) |  | 6.5 (0.9) | 6.2 (s) |
|  | 5.66 (1.60) | 5.58-5.66 (0.86) | uracyl | 5.25 (0.45) | $4.70^{c c}$ |
|  | 5.84 (0.06) | 6.28 (0.84) |  | 6.14 (0.20) | 5.10 |
| quinoxaline | 4.34 (0.02) | 3.90-3.95 (0.13) ${ }^{\text {b,f,j }}$ |  | 6.34 (0.35) | 6.10 |
|  | 4.74 (0.17) | 4.20-4.34 (0.13) |  | 6.81 (0.36) | 6.80 |
|  | 5.74 (1.73) | 5.30-5.44 (0.30) | cytosine | 4.43 (0.17) | $4.50{ }^{c c}$ |
|  | 5.69 (0.69) | 5.88-5.99 |  | 5.32 (0.07) | 5.20 |
|  | 6.31 (0.10) | 5.36 (0.61) |  | 6.16 (1.32) | 6.10 |
| sym-triazine tetrazine | 5.65 (0.40) | ${ }_{\text {5.45-5.70 }}{ }^{\text {b, }}$, e,n |  | 6.68 (0.23) | 6.70 |
|  | 5.55 (0.04) | : $5.01^{0, u, w}$ | guanine | 4.23 (0.54) | $4.50{ }^{\text {cc }}$ |
|  | 5.84 (0.32) | 0.47 |  | 5.02 (0.33) | 5.20 |
| pyrrole | 5.73 (0.36) | $5.70{ }^{\text {g }}$ |  | 6.01 (0.09) | 6.00 |
|  | 6.07 (0.00) | (i.50 |  | 6.18 (0.00) | 6.60 |
|  | 7.47 (0.90) | 7.10 - 7 , | adenine | 4.30 (0.26) | $4.80{ }^{c c}$ |
| indole | 4.50 (0.05) | 4.21-4.27 ${ }^{j, x}$ |  | 4.78 (0.20) | 5.90 |
|  | 5.00 (0.29) | 4.40-5.71 |  | 5.69 (0.96) | 6.70 |
|  | 5.76 (0.84) | 5.55-5.78 |  |  |  |
|  | 6.17 (0.81) | 5.69 |  |  |  |
|  | 6.16 (0.12) | 6.44 |  |  |  |

[^4]concern is in obtaining reliable potential surfaces, we gave a relatively small weight to the dipole moments in the least-squares fitting procedure. Yet, the agreement is similar to that obtained in all-valence-electron calculations. ${ }^{16}$
(e) Equilibrium Geometries. The refined calculated equilibrium geometries and the corresponding observed geometries are given
in Table IX. The table includes only representative samples from each class of molecules. In general, the agreement is reasonable.
(f) Vibrational Frequencies. The least-squares refinement procedure includes the vibrational frequencies of pyridine and pyrrole. The refined calculated frequencies and the corresponding observed results are given in Table X. We will mention here

Table VII. Calculated and Observed Automization Energies (in $\mathrm{kcal} / \mathrm{mol}$ ) lncluded in the Optimization of the
QCFF/PI Parameters ${ }^{i}$

| compound | calcd | obsd |
| :--- | ---: | :--- |
| pyridine | 1197.6 | $1194.3^{a}$ |
| quinoline | 1965.1 | $1964.3^{b}$ |
| pyrimidine | 1070.3 | $1071.0^{c}$ |
| pyrazine | 1072.0 | $1070.9^{c}$ |
| pyridazine | 1061.0 | $1051.3^{c}$ |
| sym-triazine | 970.1 | $965.7^{f}$ |
| pyrrole | 1027.0 | $1031.2^{d}$ |
| carbazole | 2584.0 | $2580.0^{d}$ |
| indole | 1803.0 | $1800.1^{d}$ |
| imidazole | 908.0 | $914.6^{e}$ |
| pyrazole | 902.3 | $905.3^{e}$ |
| aniline | 151.7 | $1481.0^{h}$ |
| formaldehyde | 369.3 | $364.6^{g}$ |
| actaldehyde | 641.0 | $649.4, g^{g} 695.3^{f}$ |
| acetone | 914.1 | $936.7^{g}$ |
| acrolein | 793.4 | $805.9^{f}$ |
| glyoxal | 620.1 | $615.1^{g}$ |
| $p$-benzoquinone | 1375.5 | $1382.2^{b}$ |
| benzaldehyde | 1569.7 | $1574.3^{h}$ |
| formainide | 520.0 | $544.2, f^{f} 561.3^{f}$ |

a J. D. Cox, A. R. Challoner, and A. R. Meetham, J. Chem. Soc., 265 (1954). b G. W. Wheland, "Resonance in Organic Chemistry", Wiley, New York, 1955. ${ }^{c}$ J. Tjebbes, Acta Chem. Scand., 16, 916 (1962). ${ }^{d}$ H. Zimmerman and H. Geisenfender, Z. Elektrochem., 65, 368 (1961). ${ }^{e}$ Based on heat of combustion given by Zimmerman and Geisenfender, footnote $d$. ${ }^{f}$ Based on heat of formation at $25^{\circ} \mathrm{C}$ given by G. C. Sinke, "The Chemical Thermodynamics of Organic Compounds", Wiley, New York, 1969. g Based on the selected values of heat formation at $25^{\circ} \mathrm{C}$ given by J. D. Cox and G. Pilcher, "Thermochemistry of Organic and Organometallic Compounds", Academic Press, New York, 1970. ${ }^{h}$ S. W. Benson, J. Chem. Educ., 42, 502 (1965). 'i All the values reported are in $\mathrm{kcal} / \mathrm{mol}$ and the calculated values were corrected for the vibrational enthalpies at $25^{\circ} \mathrm{C}$.
several points about the assignment: the vibrational spectra of pyridine was assigned previously by Corrsin et al., ${ }^{25}$ Kline and Turkevich, ${ }^{26}$ Wilmshurt and Bernstein, ${ }^{27}$ and McCullough et al. ${ }^{28}$ For some frequencies the assignments are strikingly different. The QCFF/PI offers a possibility to reach more unique assignments since (a) the force constants are consistent with many properties and not only vibrational spectra, and (b) the calculation involves evaluation of infrared intensities. ${ }^{20}$ This allows one to assign frequencies not only on the basis of symmetry but also on the basis of the agreement between the calculated and observed intensities. For the $\mathrm{A}_{1}$ class the present assignment is consistent with that of Corrsin et al. except the line at $1139 \mathrm{~cm}^{-1}$ which is taken from Kline and Turkevich. ${ }^{26}$ The assignment of this line is based on both the calculated frequency and the calculated intensity. The assignement of the $B_{1}$ and $A_{2}$ classes is the same as that of Corrsin et al. In the $\mathrm{B}_{2}$ class the assignment of Corrsin et al. was modified where the $1043-\mathrm{cm}^{-1}$ frequency (observed in the Raman spectrum of ref 25) was assigned as the highest frequency in this class. The very weak line at $675 \mathrm{~cm}^{-1}$ which is assigned by Corrsin et al. as a $B_{2}$ line might be a hot band ( $1375-703=672 \mathrm{~cm}^{-1}$ ), since no weak intensity $B_{2}$ line is calculated for this region.

The vibrational spectrum of pyrrole was assigned by Lord and Miller, ${ }^{29}$ Stern, ${ }^{30}$ and Mirone. ${ }^{31}$ There is significant descrepancy between the different assignments. Our assignment (based on calculated intensities, calculated frequencies, and examination of the deuterium effect) is somwhat different from that of ref 29.
(25) L. Corrsin, B. J. Fox, and R. C. Lord, J. Chem. Phys., 21, 1170 (1953).
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T. C. Kincheloe, and G. Waddington, J. Am. Chem. Soc, 79, 4289 (1957).
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(30) A. Stern and K. Thalmayer, Z. Phys. Chem. Abt. B, 31, 403 (1936).
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Table VIII. Calculated and Observed Dipole Moments (in Debye) Included in the Optimization of the QCFF/PI Parameters

| compound | calcd | obsd |
| :--- | :--- | :--- |
| pyridine | 1.90 | $2.25^{a}$ |
| quinoline | 1.86 | $2.31^{a}$ |
| isoquinoline | 2.16 | $2.57^{a}$ |
| pyrimidine | 1.76 | $2.44^{a}$ |
| pyridazine | 3.82 | $3.97^{b}$ |
| pyrrole | 1.81 | $1.84^{a}$ |
| indole | 1.74 | $2.05^{c}$ |
| carbazole | 1.21 | $2.11^{a}$ |
| imidazole | 3.88 | $3.80^{j}$ |
| pyrazole | 2.64 | $2.21^{a}$ |
| formaldehyde | 2.95 | $2.34^{d}$ |
| acetaldehyde | 3.00 | $2.69^{e}$ |
| acetone | 3.60 | $2.90^{g}$ |
| acrolein | 3.51 | $3.11^{f}$ |
| o-benzoquinone | 5.41 | $5.1^{h}$ |
| uracyl | 4.47 | $5.18^{k}$ |
| adenine | 1.63 | $2.88^{m}$ |
| benzaldehyde | 3.55 | $3.0^{a}$ |
| aniline | 1.73 | $1.53^{i}$ |
| purine | 5.4 | $4.32^{l}$ |
| cytosine | 7.7 | $8.0^{m}$ |

${ }^{a}$ A. L. McClellan, in "Tables of Experimental Dipole Moments", W. H. Freeman, San Francisco, 1963. b W. C. Schneider, J. Am. Chem. Soc., 70, 627 (1948). c E. G. Cowley and J. R. Partington, J. Chem. Soc., 47 (1936). d J. N. Shoolery and A. H. Sharbaugh, Phys. Rev., 82, 95 (1951). ${ }^{e}$ R. Kilb, C. C. Lin, and E. B. Wilson, J. Chem. Phys. 26, 1695 (1957). ${ }^{f}$ R. Wagner, J. Fine, J. W. Simmons, and J. H. Goldstein, J. Chem. Phys., 26, 634 (1957). ${ }^{g}$ J. D. Swalen and C. C. Costain, J. Chem. Phys., 31, 1562 (1959). ${ }^{h}$ S. Nagakura and A. Kuboyana, J. Am. Chem. Soc., 76, 1003 (1954). ${ }^{i}$ C. W. N. Cumper, R. F. A. Ginınan, D. G. Redford, and A. I. Vogel, J. Chem. Soc., 1731 (1963). ${ }^{j}$ K. Hoffmann in "Imidazole and Its Derivatives", Part I, Interscience, New York, 1953. ${ }^{k}$ P. Mauret and J. P. Fayet, C. R. Acad. Sci., Ser. C, 264, 2081 (1967). ${ }^{l}$ E. D. Bergmann and F. Weiler in "The Purines Theory and Experiment"', Vol. 4, Eds., E. D. Bergmann and B. Pullmann. The Jerusalem Symposia on Quantum Chemistry and Biochemistry, p 21. ${ }^{m} \mathrm{H}$. DeVoe and I. Tinoco, (estimated from the dipoles of the corresponding methylated compounds), J. Mol. Biol., 4, 500 (1962).

The $1146-\mathrm{cm}^{-1}$ mode, assigned in ref 29 to both $\mathrm{A}_{1}$ and $\mathrm{B}_{1}$ classes, is not included in the $A_{1}$ class. The very weak Raman line at 1237 $\mathrm{cm}^{-1}$, assigned by ref 29 as an $\mathrm{A}_{1}$ mode, is not included in the $\mathrm{A}_{1}$ class in view of its uncertain polarization. The previous assignment of the strong line at $1146 \mathrm{~cm}^{-1}$ as a $\mathrm{B}_{1} \mathrm{~N}-\mathrm{H}$ bending mode $\mathrm{e}^{29}$ was found to be inconsistent with comparison of the calculated and observed frequency shift upon deuteration of the pyrrole nitrogen. The $\mathrm{B}_{1} \mathrm{~N}$-H was calculated in this work as a very weak line at $1364 \mathrm{~cm}^{-1}$ which cannot be assigned to the strong line at 1146 $\mathrm{cm}^{-1}$.

## IV. Applications

The method presented in this paper was used extensively in the past few years. Here we list several types of studies where the method was found to be particularly useful.
(a) Calculations of Resonance Raman (RR) Spectra. The calculation of ground- and excited-state equlibrium geometries and vibrational normal modes provides a powerful tool for evaluation of RR spectra of biological chromophores. ${ }^{3}$ This was demonstrated in calculations of the RR spectrum of retinal and related molecules ${ }^{32}$ and in analysis of the relation between "the spin state marker" frequencies and the heme geometry in metalloporphyrins. ${ }^{3}$ In order to demonstrate the method we present

[^5]Table IX. Calculated and Observed Geometries ${ }^{a, b}$ Included in the Optimization of the QCFF/PI Parameters

${ }^{a}$ All bond lengths in $\AA .{ }^{b}$ All angles in degrees. ${ }^{c}$ B. Bak, L. Hansen-Nygaard, and J. Rastrup-Anderson, J. Mol. Spectrosc., 2, 361 (1958). d P. S. Wheatley, Acta Crystallogr. 13, 80 (1960). ${ }^{e}$ P. S. Wheatley, Acta Crystallogr., 10, 182 (1957). $f$ D. H. Sutter and W. H. Flygare, J. Am. Chem. Soc., 91, 6895 (1969). ${ }^{\text {g S S. }}$ Martinez-Carrera, Acta Crystallogr., 20, 783 (1966). ${ }^{h}$ F. Krebs Larsen, M. S. Lehmann, I. Sotofte, and S. E. Rasmussen, Acta Chem. Scand., 84, 3248 (1970). ${ }^{i}$ K. Takagi and T. Oka, J. Phys. Soc. Jpn., 18, 1174 (1963). jR. Kilb, C. C. Lin, and E. B. Wilson, J. Chem. Phys., 26, 1695 (1957). ${ }^{k}$ R. Nelson and L. Pierce, J. Mol. Spectrosc., 18, 344 (1965). ${ }^{l}$ J. Trotter, Acta Crystallogr., 13, 86 (1960). ${ }^{m}$ E. A. Cherniak and C. C. Costain, J. Chem. Phys., 45, 104 (1966).
in Table XI a complete normal mode analysis of a metalloporphyrin (where the metal is represented as an effective $\sigma$ charge) and compare the calculated and observed RR intensities in Figure 2. This study provides the first analysis of the out-of-plane vibrations of porphyrins. Such an analysis could not have been accomplished by conventional normal mode approaches because of the complexity of the problem and the uncertainty about the proper force constants. The RR intensities calculations were found to be of great help in the assignment of the heme vibrations.
(b) Environmental Effects. The present method allows for a convenient evaluation of environmental effects on spectroscopic

Table X. Experimental and Calculated Vibrational Frequencies Used in the Optimization of the Energy Parameters

Pyridine

|  | obsd ${ }^{\text {a }}$ | calcd |
| :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 3054 | 3093 (vw) |
|  | 3054 (s) | 3090 (vw) |
|  | 3036 (vs) | 3088 (vs) |
|  | 1580 (vs) | 1555 (s) |
|  | 1482 (vs) | 1440 (s) |
|  | 1139 (vvw) ${ }^{\text {b }}$ | 1140 (vvw) |
|  | 1068 (s) | 1047 (m) |
|  | 1029 (vs) | 1041 (s) |
|  | 992 (vs) | 994 (vs) |
|  | 605 (s) | 660 (m) |
| $\mathrm{B}_{1}$ | 3083 (vs) | 3090 (vs) |
|  | 3054 | 3087 (s) |
|  | 1572 (vs) | 1545 (w) |
|  | 1439 (vs) | 1439 (vs) |
|  | 1375 (m) | 1331 (w) |
|  | 1217 | 1219 (s) |
|  | 1148 (s) | 1142 (m) |
|  | 1068 | 1045 (w) |
|  | 652 (w) | 666 (w) |
| $\mathrm{A}_{2}$ | 981 (r) | 1017 |
|  | 886 (r) | 848 |
|  | 375 (r) | 417 |
| $\mathrm{B}_{2}$ |  |  |
|  | $942 \text { (vvw) }$ | $915 \text { (vvw) }$ |
|  | 749 (vvs) | 738 (vvs) |
|  | 703 (vvw) | 727 (vvs) |
|  | 403 (s) | 462 (m) |
|  | Pyrrole |  |
|  | obsd ${ }^{\text {d }}$ | calcd |
| $\mathrm{A}_{1}$ | 3400 | 3370 (vs) |
|  | 3133 | 3104 (m) |
|  | 3100 | 3103 (vs) |
|  | 1467 | 1470 (m) |
|  | 1418 (s) | 1461 (vs) |
|  |  | 1123 (vvw) |
|  | 1076 (vs) | 1086 (w) |
|  | 1015 (vs) | 979 (m) |
|  | 711 (vs) | 802 (vw) |
| $\mathrm{B}_{1}$ | 3133 | 3108 (w) |
|  | 3111 | 3101 (vvs) |
|  | 1530 (vs) | 1565 (s) |
|  | 1384 (m) | 1396 (m) |
|  |  | 1364 (vvw) |
|  | 1146 (s) | 1153 (w) |
|  | 1046 (vs) | 1065 (s) |
|  | 768 (s) | 782 (m) |
| $\mathrm{A}_{2}$ | $1091(\mathrm{r})^{e}$ | 1126 (r) |
|  | 868 (r) | 896 (r) |
|  | 510 (r) ${ }^{f}$ | 523 (r) |
| $\mathrm{B}_{2}$ | 1046 (m) | 1117 (w) |
|  | 880 (w) | 931 (w) |
|  | 647 (m) | 676 (vvs) |
|  | 565 (w) | 524 (w) |

${ }^{a}$ All frequencies in $\mathrm{cm}^{-1}$, intensity notation vs, $s, m, w, v w, r$ for very strong, strong, medium, weak, very weak, and only Raman active, respectively. The observed frequencies are taken from ref 25 except when indicated otherwise (also see text). ${ }^{b}$ Taken from ref 26 . ${ }^{c}$ See discussion in text. ${ }^{d}$ The observed frequencies are taken from ref 29 except when indicated; otherwise for discussion of the assignment, see text. ${ }^{e}$ Taken from ref $30 .{ }^{f}$ Estimated in ref 29 with the help of the product rule.
properties of conjugated molecules. The electrostatic potential from the charges and dipoles of the given environment is incorporated into the $\pi$-electron Hamiltonian in the same way as the electrostatic potential from the $\sigma$ charge. That is, the diagonal SCF matrix element of the isolated molecule $\left(F_{v v}\right)^{0}$ is modified by:

$$
\begin{equation*}
F_{\nu \nu}{ }^{\pi}=\left(F_{\nu \nu}{ }^{\pi}\right)^{0}-\sum_{\mathrm{C}} Q_{\mathrm{C}} e^{2} / r_{\mathrm{AC}}+\sum_{\mathrm{D}} \mu_{\mathrm{D}} \mathrm{r}_{\mathrm{AD}} / r_{\mathrm{AD}}{ }^{3} \tag{21}
\end{equation*}
$$

where the $\nu t h \pi$ orbital is located on atom A, and $Q$ and $\mu$ are

Table XI. Normal Mode Analysis of a Metalloporphyrin ${ }^{\text {a }}$

|  | calcd | obsd ${ }^{\text {b }}$ | description |  | calcd | obsd ${ }^{\text {b }}$ | description |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1 \mathrm{~g}}$ | 3106 |  | C-H |  | 373 |  | $\chi^{\chi}$ |
|  | 3105 |  | C-H |  | 172 |  | $\chi_{m}$ |
|  | 1600 | $1562^{1}$ | $\mathrm{b}-\mathrm{b}, \mathrm{m}-\mathrm{a}$ |  | 130 |  | $\chi^{\times}$ |
|  | 1395 | $1370(1425)^{2}$ | $\mathrm{a}-\mathrm{b}, \delta(\mathrm{CH})$ | $\mathrm{E}_{\mathbf{u}}$ | 3106 | $3076{ }^{4}$ | $\mathrm{C}-\mathrm{H}$ |
|  | 1363 | $1336{ }^{1}$ | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{m}$ |  | 3103 | $3062{ }^{4}$ | C-H |
|  | 1106 | $998{ }^{1}$ | $\delta(\mathrm{CH})$ |  | 3082 | $3062^{4}$ | $\mathrm{C}-\mathrm{H}$ |
|  | 698 | $732^{1}$ | $\delta$ (ma) |  | 1652 | $1690^{4}$ | m-a, a-b |
|  | 619 | $674{ }^{2}$ | $\mathrm{a}-\mathrm{N}, \delta$ (ring) |  | 1575 | $1550{ }^{4}$ | $\mathrm{b}-\mathrm{b}, \mathrm{m}-\mathrm{a}$ |
|  | 316 | $368{ }^{1}$ | $\mathrm{N}-\mathrm{M}$ |  | 1446 | $1520^{4}$ | $\mathrm{a}-\mathrm{b}, \delta(\mathrm{CH})$ |
| $\mathrm{A}_{2 \mathrm{~g}}$ | 3103 |  | $\mathrm{C}-\mathrm{H}$ |  | 1400 | $1389{ }^{4}$ | $\mathrm{a}-\mathrm{N}, \mathrm{a}-\mathrm{m}, \delta(\mathrm{CH})$ |
|  | 1610 | $1587{ }^{1}$ | a-m |  | 1340 | $1302^{4}$ | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{N}$ |
|  | 1409 | $1420{ }^{2}$ | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{m}$ |  | 1310 |  | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{b}$ |
|  | 1360 | $1322^{1}$ | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{N}$ |  | 1180 | 11554 | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{b}$ |
|  | 1180 | $1143{ }^{1}$ | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{b}$ |  | 1103 | 9934 | $\delta(\mathrm{CH})$ |
|  | 820 | $802^{1}$ | $\delta$ (ring) |  | 833 | $843^{4}$ | $\delta$ (ring) |
|  | 695 | $604{ }^{3}$ | $\delta$ (ring) | $\mathrm{E}_{\mathbf{u}}$ | 770 | $740{ }^{4}$ | $\mathrm{a}-\mathrm{N}, \delta$ (ma) |
|  | 363 |  | $\delta(\mathrm{Na}), \delta$ (ring) |  | 645 |  | $\mathrm{a}-\mathrm{N}, \delta(\mathrm{Na})$ |
| $\mathrm{B}_{1 \mathrm{~g}}$ | 3082 |  | $\mathrm{C}-\mathrm{H}$ |  | 631 | 7034 | $\mathrm{a}-\mathrm{N}, \delta$ (ring) |
|  | 1677 | $1631{ }^{1}$ | $a-m$ |  | 460 | $385{ }^{4}$ | $\mathrm{N}-\mathrm{M}$ |
|  | 1545 | $1547^{2}(1498)^{1}$ | b-b |  | 330 | $348{ }^{4}$ | $\delta$ (ring) |
|  | 1334 | $1352^{1}$ | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{N}$ |  | 226 | $202^{4}$ | $\delta(\mathrm{Na})$ |
|  | 1207 | $1180^{1}$ | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{b}$ | $\mathrm{A}_{2 \mathrm{~L}}$ | 852 | 860 | $\chi_{\text {H }}(\mathrm{m})$ |
|  | 1096 | $1059^{1}$ | $\delta(\mathrm{CH})$ |  | 855 |  | $\chi^{\text {H }}$ |
|  | 780 | $760^{2}$ | $\delta$ (ring) |  | 788 | 760 | XC |
|  | 559 |  | $\delta$ (Na) |  | 581 | $699^{4}$ | XC |
|  | 238 | $206{ }^{1}$ | $\mathrm{N}-\mathrm{M}$ |  | 278 |  | $\chi_{\text {m }}$ |
| $\mathrm{B}_{2 \mathrm{~g}}$ | 3107 |  | C-H |  | 153 | $167^{4}$ | $\chi_{M}$ |
|  | 3082 |  | $\mathrm{C}-\mathrm{H}$ |  | 54 |  | $\mathrm{X}_{\mathrm{M}}$ |
|  | 1470 | $1498{ }^{1}$ | a-b | $\mathrm{A}_{1 \mathrm{u}}$ | 579 |  | $\chi^{1}$ |
|  | 1422 | $1380{ }^{1}$ | $\mathrm{a}-\mathrm{N}, \mathrm{a}-\mathrm{m}$ |  | 290 |  | $\chi^{+}$ |
|  | 1243 | $1288{ }^{1}$ | $\delta(\mathrm{CH}), \mathrm{a}-\mathrm{m}, \mathrm{a}-\mathrm{b}$ | $\mathrm{B}_{14}$ | 1063 |  | $\chi^{+}$ |
|  | 858 | $853^{1}$ | $\delta$ (ring) |  | 788 |  | $\chi^{\text {H }}$ |
|  | 650 |  | $\mathrm{a}-\mathrm{N}, \delta$ (ring) |  | 594 |  | $\chi_{\text {C }}$ |
|  | 426 | $426^{1}$ | $\delta$ (ring) |  | 414 |  | $\chi_{\text {m }}$ |
|  | 107 |  | $\delta(\mathrm{aN})$ |  | 77 |  |  |
| $\mathrm{E}_{\mathrm{g}}$ | 1064 |  | $\chi^{\text {H }}$ | $\mathrm{B}_{2 \mathrm{u}}$ | 1062 |  | $\chi{ }_{H}$ |
|  | 851 |  | $\mathrm{X}_{\mathrm{H}(\mathrm{m})}$ |  | 541 |  | $\chi_{\text {c }}$ |
|  | 787 |  | $\chi_{\text {H }}$ |  | 146 |  | $\chi_{N}$ |
|  | 591 556 |  | $\chi^{\chi_{C}}$ |  | 55 |  | $\chi_{\text {a }}$ |
|  | 556 |  | $\chi_{\text {c }}$ |  |  |  |  |

${ }^{a}$ The assignment is based on comparing calculated and observed RR and IR intensities. Frequencies in $\mathrm{cm}^{-1}$; the atom types $a, b, m, N$, and M designate respectively the carbon atoms bound to nitrogen, the pyrrole carbon atoms which are not bound to N , the meso atoms, the nitrogen atoms, and the metal atom. X-Y designate stretching frequency of an $\mathrm{X}-\mathrm{Y}$ bond. $\delta$ designates bending frequency, $\delta$ (ring) designates bending of the whole porphyrin ring. $\chi_{y}$ designates out of plane deformation around atom $y$. $b$ The observed frequencies are taken from ref $33,34,35$, and 36 which are indicated here as $1,2,3$, and 4 , respectively.
the partial charges and induced dipoles of the atoms of the environment molecules. ${ }^{37,38}$ If the environment molecules are represented by permanent dipoles (rather than point charges), they are included in the last term of this equation. The practical evaluation of the solvent dipoles for polar and nonpolar solvents was described in detail previously. ${ }^{37,38}$

As a demonstration of evaluation of environmental effects, we describe here calculations of the spectral shifts in hemoglobin upon allosteric transition; the hemoglobin molecule is characterized by two tertiary structures: high oxygen affinity form, (r) and low oxygen affinity form ( t ). Transition of the ligated hemoglobin from $r$ to $t$ shifts the absorption spectrum of the $Q$ and Soret(s) bands to the red by $40-5 \mathrm{~cm}^{-1}$. 39,40 In order to calculate the spectral shifts we evaluate the electrostatic potential from the protein at the heme atoms using the X-ray structures of methemoglobin and deoxyhemoglobin as representing the $r$ and $t$ states

[^6]Table XII. Calculated Shifts of the Q and S Bands of the Porphyrins upon $r \rightarrow t$ Transition in Hemoglobin

| protein electrostatic <br> potential at the heme | band | $\Delta E, \mathrm{~cm}^{-1}$ | $f$ |
| :---: | :---: | :---: | :---: |
| potential from the t state | Q | 15533 | 0.226 |
|  | S | 28333 | 3.040 |
| potential from the r state | Q | 15566 | 0.403 |
|  | S | 28370 | 3.057 |

${ }^{a} \Delta E$ and $f$ are the transition energy and oscillator strength of the indicated electronic transitions. The degenerate $S$ and $Q$ transitions are split by the protein electric field and the $\Delta E$ presented here correspond to the average value of the split transition energies.
(see ref 41 for more details). The potentials from the protein tertiary structures, which are presented in Figure 4 of ref 41, were introduced into the SCF $\pi$-electron Hamiltonian of the heme atoms using eq 21. The calculated spectral shifts are compared to the observed one in Table XII. As seen from the table, the calculation reproduces red shifts of the same order as the observed shifts.
(c) Protonation Effects. Many biological processes involve protonated chromophores. These include protonated Schiff bases of retinal in rhodopsin and bacteriorhodopsin. ${ }^{6}$ Calculations of

[^7]

Figure 2. Calculated and observed ${ }^{33}$ perpendicular and parallel contributions to the RR spectrum of the Q band of a typical porphyrin: (a) the calculated contribution of the A term to $I_{\| j}$; b) the calculated B term contribution to $I_{I I}$ of the $\mathrm{B}_{1 \mathrm{~g}}$ and $\mathrm{A}_{1 g}$ modes (the calculated intensities of the $\mathrm{B}_{28}$ modes are much smaller); (c) the observed $I_{\|}$of Cu -porphine ${ }^{33}$ (the line at $1498 \mathrm{~cm}^{-1}$ is much weaker in other porphyrins where other strong $\mathrm{B}_{8}$ lines appear at $\sim 1550 \mathrm{~cm}^{-1}$ ); (d) the observed $I_{\perp}$ for $\mathrm{Cu}-$ porphine; (e) the calculated B term contribution to $I_{\perp}$.
the effect of protonation on absorption and resonance Raman spectra are straightforward. The proton charge is distributed as a $\sigma$ charge between the proton and the atoms at the protonation site (the $\sigma$ charge distribution can be determined by all-va-lence-electron calculations). This $\sigma$ charge is treated simply as any other $\sigma$ charge by eq 13 , including the modifications of $\bar{W}_{\mu}$ by adding the $Q_{\mu}{ }^{\sigma} \gamma_{\mathrm{AA}}$ term associated with the new $\sigma$ charge. This type of calculation was used extensively in studying the effect of protonation on the spectrum of protonated Schiff bases of retinal. ${ }^{38,42}$
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## V. Concluding Remarks

This paper presents an extension of the QCFF/PI method to heteroatom-containing conjugated molecules. The extension involves incorporation of the $\sigma$ charges into the $\pi$-electron Hamiltonian by treating formally the SCF equation as an all-va-lence-electron problem, assuming zero $\sigma-\pi$ overlap, and then fixing the $\sigma$-electron bond orders. The resulting submatrix of the $\pi$ orbitals is the one used for the $\pi$-electron calculations. The incorporation of the potential from the $\sigma$ atom of a given molecule in its $\pi$-electron Hamiltonian is generalized to incorporate potential from charges and induced dipoles of neighboring molecules (eq 21). This offers a simple and reliable way of incorporating environmental effects in calculations of spectroscopical properties of biological chromophores. This can be applied in detailed studies of spectral shifts of substrates or prosthetic groups in active sites of proteins with known X-ray structure. Such a study is demonstrated here for the spectral shift of hemoglobin upon $r \rightarrow t$ transition. Other related studies are now underway in our laboratory.
Incorporation of external potentials from neighboring molecules in $\pi$-electron calculations might be crucial for consistent study of the important class of charge-transfer crystals of TCNQ and related molecules. ${ }^{43}$ In such cases, the present method can be implemented in the Molecular Crystal Analysis (MCA) program. ${ }^{44,45}$

The extension of the QCFF/PI method to conjugated molecules allows for interpretation of RR spectra of most biological chromophores. This is demonstrated here by the normal mode analysis of a porphyrin. Other studies are reported in ref 3. The present approach also offers the possibility of studying environmental effects on RR spectrum of biological chrmophores. Such studies may be done by simply using eq 21 in calculating RR spectrum of chromophores in protein active sites.

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# Study of Additivity of Correlation and Polarization Effects in Relative Energies 

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#### Abstract

Calculations are reported at the $6-31 \mathrm{G}$ and $6-31 \mathrm{G}^{* *}$ level with and without inclusion of all double excitations (CID/6-31G and CID/6-31G**). The relative effects of polarization or correlation are compared with calculations involving polarization and correlation. The error in additivity is $\sim 2 \mathrm{kcal} / \mathrm{mol}$ for systems involving simple geometric changes and $\sim 4$ $\mathrm{kcal} / \mathrm{mol}$ for systems involving dimerization and molecular combination. These results are especially interesting in electron-deficient systems involving relatively large corrections due to differences in classical vs. nonclassical bonding. We emphasize that additivity in total energies is neither expected nor observed.


As computers become more efficient and computer methods more effective, the problem of correlation is becoming more tractable. Several prominent groups in theoretical chemistry have recently ${ }^{1,2}$ solved the problem of determining analytical gradients

[^8]from correlated wave functions. Other groups are concerned with CI effects and geometries ${ }^{3}$ notably Kutzelnigg ${ }^{4-9}$ and Meyer. ${ }^{10,11}$
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